A RANDOM-DIFFERENCE SERIES FOR USE IN THE ANALYSIS OF TIME SERIES

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It has several times been noted that time series commonly possess in many respects the characteristics of series of cumulated random numbers. The separate items in such time series are by no means random in character, but the changes between successive items tend to be largely random. This characteristic has been noted conspicuously in sensitive commodity prices. On the basis of the differences between chain and fixed-base index numbers King has concluded that stock prices resemble cumulations of purely random changes even more strongly than do commodity prices.[1]

The fact that series commonly used as indexes of business activity closely resemble series obtainable by cumulating random numbers has given support to the theory that so-called business cycles result in large degree from cumulative effects of independent random influences bearing on the business situation—some favorably, some unfavorably.[2]

In the discussion which follows, a series such as may be obtained by cumulating random numbers will for brevity and clarity be called usually a *random-difference series*, since it is the first differences of the series and not the items of the series itself which are random. The natural alternative term of *cumulated random series* is subject to misinterpretation as describing a series that is itself random.

Economic theory has fallen far short of recognizing the full implications of the resemblance of many economic time series to random-difference series; and methods of statistical analysis in general use have given these implications virtually no recognition. Economic theories and the techniques that have been employed in analysis of time series generally deal in terms of norms and of deviations therefrom. The norms may be regarded as constants (as is common in economic theory) or they may be regarded as changing progressively (represented in statistical practice by trend lines). The deviations from norms or from trends are commonly regarded as having one of three characteristics: (a) that of random deviation, each item independent of all others; (b) that of cycles, either regular in periodicity and amplitude or irregular, but in any case with a definite tendency for deviation in one direction

[1] W. I. King, *Index Numbers Elucidated*, 1930, pp. 99, 102.

[2] Apparently E. E. Slutsky's *The Summation of Random Causes as the Source of Cyclic Processes*, 1927, deserves chief mention in this connection.

to be followed after an interval by deviation in the other (not by accident but in consequence of a specific reaction tendency); or (c) some combination of random deviations with a cyclical tendency, or with several cyclical tendencies of differing period and amplitude.

These concepts are inappropriate and misleading when applied to cases in which the dominant tendency is for the effects of successive events to be independent and cumulative. An outstanding characteristic of a series of this type is that its *changes* are largely random and unpredictable. In a simple and regular cyclical series changes are accurately predictable. Even in a purely random series (one of random deviations from a norm) in which individual values are unpredictable, *changes* are predictable with considerable accuracy—the coefficient of correlation between absolute values in a random series and immediate subsequent changes is $r = -.71$.[1] In a series characterized by primarily random *changes*, however, absolute values of immediately subsequent items are predictable with an accuracy that for many purposes may be regarded as very satisfactory, but subsequent changes are largely unpredictable.[2]

Brief study of the charts which follow will show that in a series composed of purely random changes (a strict random-difference series) conspicuous trends will be found. Such "trends," however, must be regarded merely as generalized descriptions of the course of the series over a certain period, not as norms, nor as bases for predicting the future course of the series over even the briefest subsequent period. In such a series also, frequent movements of indubitable cyclical conformation occur, but they cannot be regarded as evidence of true cycles —even irregular cycles—unless that term be divorced from its usual connotation of movement that tends to generate, or at least to reflect influences that tend to generate, reverse movement.

Few if any time series will be encountered that reflect in pure form the condition of strictly random changes, as few if any time series are

[1] If x_i represent any value in a random series, in terms of deviations from the mean, and $(x_{i+1}-x_i)$ the immediate subsequent change, the coefficient of correlation between the two is given by

$$r = \frac{\Sigma x_i(x_{i+1}-x_i)}{\sqrt{\Sigma x^2_i . \Sigma(x_{i+1}-x_i)^2}}.$$

Substituting in the expansion of this expression the most probable values: $\Sigma x_i x_{i+1} = 0$ (since x_i and x_{i+i} tend to be uncorrelated) and $\Sigma x^2_{i+1} = \Sigma x^2_i$ (since each tends to equal n times the squared standard deviation of the population from which the sample is drawn), we obtain as the expected value of the correlation coefficient:

$$r = -\frac{\Sigma x^2_i}{\sqrt{\Sigma x^2_i . 2\Sigma x^2_i}} = -\frac{1}{\sqrt{2}} = -.7071.$$

[2] Knowing the closing price of a stock or of a speculative commodity such as wheat, any one can forecast the closing price on the next day within a range of error that for many purposes may be regarded as inconsequential; but few students of these markets believe it possible to forecast the next day's price change with any considerable degree of accuracy.

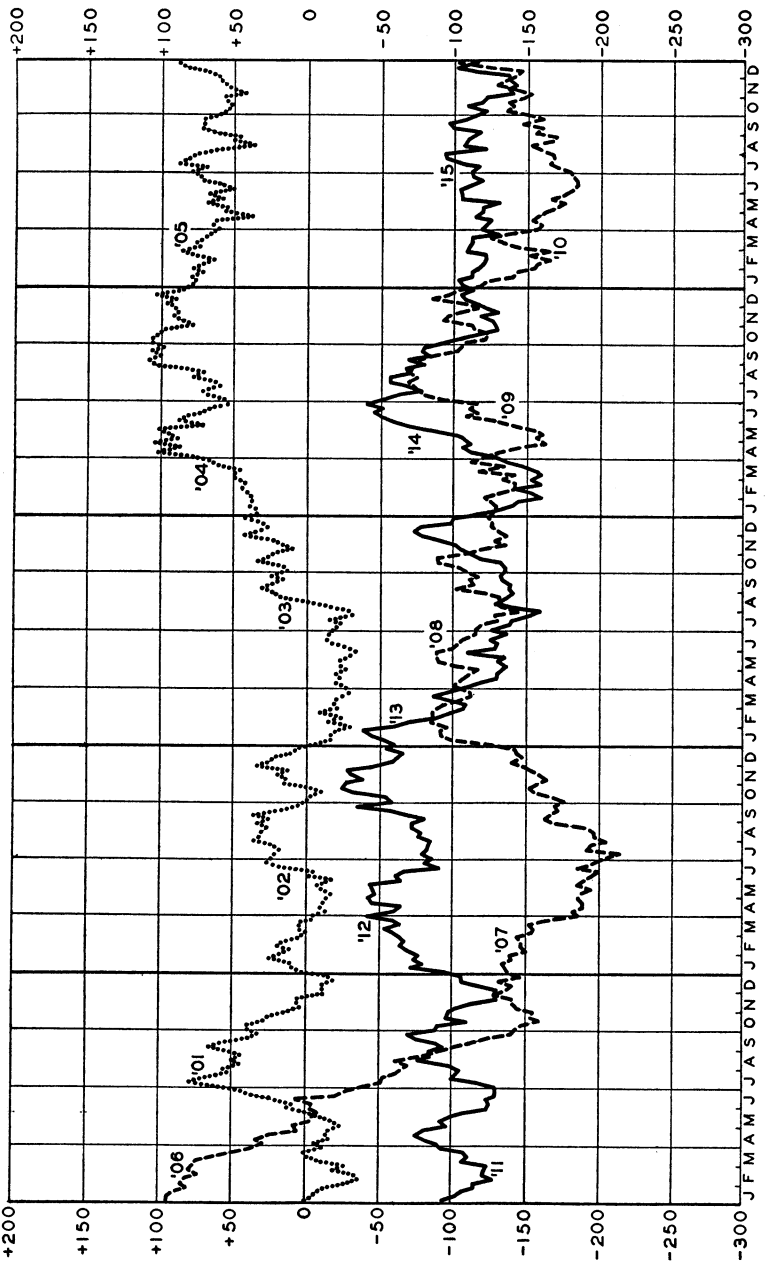


CHART IA—RANDOM-DIFFERENCE EXPERIMENTAL TIME SERIES, PART 1

CHART IB—RANDOM-DIFFERENCE EXPERIMENTAL TIME SERIES, PART 2

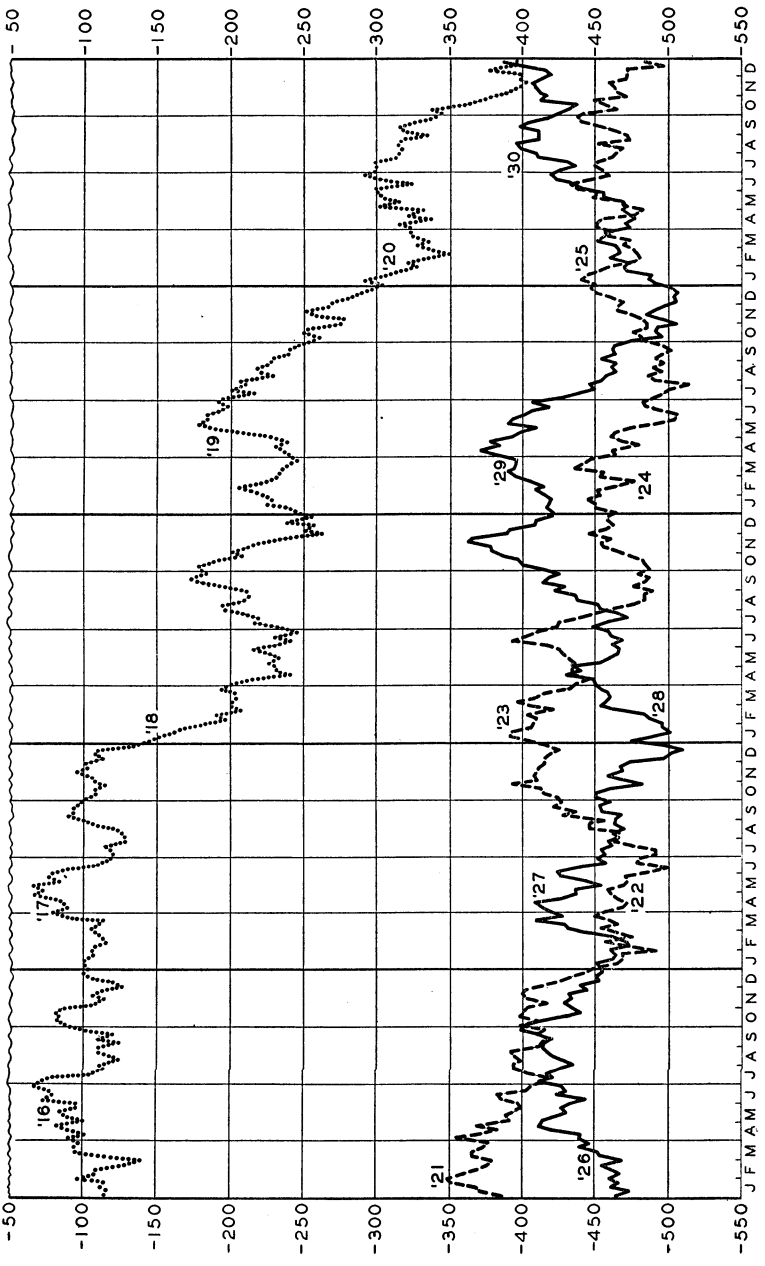
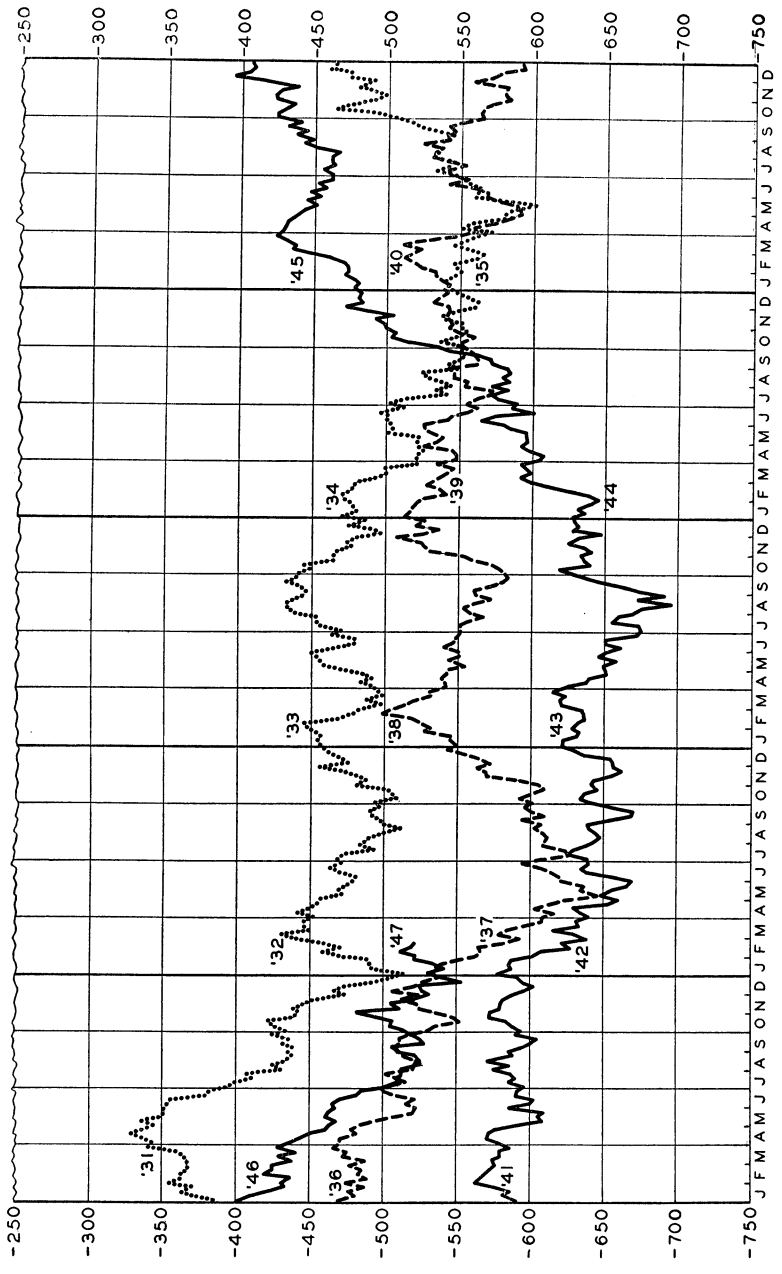


CHART IC—RANDOM-DIFFERENCE EXPERIMENTAL TIME SERIES, PART 3



encountered in practice that offer pure examples of random deviations, or pure examples of cyclical movement such as appears in a simple or complex harmonic series. Generally in time series a combination of characteristics is reflected, and circumstances must determine whether the series shall be treated as though it possessed only its dominant characteristic, or treated in such a way as to take account of all its significant characteristics. But the characteristic of random changes is present in such important degree in sensitive price series, and probably in many other economic series, as to deserve serious consideration.

For the purpose of testing whether or to what extent any particular time series possesses the characteristics of a random-difference series, it is desirable to have available for comparison an extended series known to possess those characteristics and those alone. In the statistical analysis of time series showing such characteristics in important degree, it is essential for certain purposes to have such a standard series to provide a basis for statistical tests. The remainder of this paper is devoted to presenting such a standard series, graphically and in tabular form, and to suggesting uses to which it may profitably be put in statistical analysis.

A STANDARD RANDOM-DIFFERENCE SERIES

In a random-difference series designed for serious use as a type standard in statistical analysis, it is essential above all that the first differences of the series be as nearly as possibly strictly random. The standard deviation of the population of which the random first differences constitute a sample should be accurately known. Because of the superior generality of the "normal" frequency distribution, it is preferable that the population providing the random first differences should be one of "normal" frequency distribution.

When the "random" numbers are obtained by assigning numbers to the letters of the alphabet and "drawing" by taking them in the order in which they appear in sentences, as has been done by some writers for illustration, the effect is to draw from a universe of very peculiar frequency distribution and to draw in a manner which introduces certain other peculiarities. For example, the letters *o* and *s* will appear with much greater frequency than the intervening letter *q*. Also there will tend to be peculiar correlations in the order of drawing, such as those consequent on the substantial tendency for vowels and consonants to alternate; and frequent recurrence of certain number series, such as those corresponding to "ing" and "tion." It is important to avoid all such characteristics in the random series which on successive cumulation will provide the desired random-difference series.

The accompanying series is derived basically from the tables of Tippett's *Random Sampling Numbers*.[1] These tables were prepared in the Biometric Laboratory at the suggestion of its Director, Karl Pearson, with the object of providing an improved basis for random sampling experiments and to replace the conventional methods which ideally should give truly random samples, but which in practice did not. In the Foreword Pearson states: "In short, tickets and cards, balls and beads fail in large scale random sampling tests; it is as difficult to get artificially true random samples as it is to sample effectively a cargo of coal or of barley."

Tippett's numbers as they stand constitute a large sample (or by variation in the manner in which the numbers are read, a number of samples) from a population having a rectangular distribution. By appropriate transformations of the numbers in the sample, they may be made to form a random sample from a distribution of any desired form. For the present use they have been converted to form a sample from a normal distribution with S.D. = 10,[2] following the method described by Pearson on page iv of *Random Sampling Numbers*. The following reproduction of portions of the table prepared for use in making the transformation illustrates the method and indicates the treatment of problems of detail encountered.

Number in Tippett's tables	*Corresponding number in normal distribution*	*Frequency per 10,000 in population*
0000	(discarded)	...
0001	—39	1
0002	—35	1
0003	—34	1
0004–0005	—33	2
....	....	...
....	....	...
4404–4800	— 1	397
4801–5199	0	399
5200–5596	+ 1	397
....	....	...
....	....	...
9995–9996	+33	2
9997	+34	1
9998	+35	1
9999	+39	1

[1] *Tracts for Computers*, No. XV, Cambridge University Press, London, 1927.

[2] The standard deviation has been taken as 10 rather than as unity to avoid the inconvenience of decimals. To convert the cumulated series to one based on changes drawn from a population of unit standard deviation it is of course necessary only to insert a decimal point before the final digit of each figure.

TABLE I

EXPERIMENTAL TIME SERIES:—CUMULATIVE TOTALS OF 2399 RANDOM NUMBERS

- 5	+ 14	+ 44	+ 55	-158	-111	- 77	-121	- 84	-114	-112	-105	-232	-260	-374	-456
- 7	+ 28	+ 33	+ 68	-153	-102	- 84	-121	- 85	-128	-118	-113	-227	-250	-376	-428
- 13	+ 24	+ 27	+ 58	-155	-103	-103	-123	- 77	-139	-112	-126	-233	-253	-355	-436
- 29	+ 21	+ 30	+ 67	-143	-107	-105	-107	- 78	-144	-112	-119	-229	-275	-360	-421
- 35	+ 18	+ 43	+ 52	-140	-112	-108	-110	- 73	-159	-117	-105	-215	-277	-383	-426
- 33	+ 24	+ 33	+ 59	-140	-117	-121	-108	- 73	-152	-106	-100	-229	-257	-369	-424
- 19	+ 37	+ 34	+ 71	-128	-103	-122	- 94	- 81	-142	- 95	-103	-241	-251	-391	-414
- 27	+ 31	+ 41	+ 75	-137	- 90	-114	- 90	- 71	-159	- 94	-100	-230	-267	-389	-412
- 10	+ 31	+ 38	+ 79	-140	- 88	-113	- 82	- 57	-153	-121	-101	-247	-270	-394	-393
- 1	+ 27	+ 38	+ 70	-132	- 88	- 94	- 75	- 35	-159	-107	-107	-235	-281	-398	-410
+ 1	+ 33	+ 41	+ 89	-147	- 99	- 96	- 79	- 58	-156	-108	-105	-217	-288	-397	-408
- 12	+ 26	+ 45	+ 83	-137	-104	-105	- 97	- 55	-146	-111	-107	-219	-299	-386	-410
- 7	+ 36	+ 42	+ 77	-135	-107	-116	- 92	- 32	-137	-117	-116	-210	-304	-384	-412
- 15	+ 16	+ 48	+ 61	-133	-115	-108	- 98	- 24	-129	- 99	-113	-196	-292	-403	-415
- 14	+ 7	+ 49	+ 36	-139	-115	- 85	-106	- 26	-112	- 96	-108	-195	-305	-406	-416
- 15	0	+ 46	+ 50	-138	-119	- 96	-124	- 37	-106	-107	-105	-209	-316	-413	-420
- 23	- 1	+ 61	+ 47	-149	-130	-103	-125	- 30	-111	-114	-110	-213	-328	-416	-425
- 18	- 9	+ 66	+ 64	-147	-128	-118	-124	- 28	-106	-121	-113	-211	-322	-402	-416
- 9	- 2	+ 82	+ 72	-147	-144	-122	-128	- 44	-103	-109	- 90	-196	-334	-394	-405
- 1	+ 15	+102	+ 71	-143	-134	-140	-129	- 58	- 84	-118	- 79	-182	-349	-394	-392
+ 14	+ 14	+ 88	+ 71	-154	-129	-142	-129	- 62	- 72	-121	- 88	-173	-337	-399	-393
+ 10	+ 19	+103	+ 61	-155	-132	-154	-117	- 65	- 59	-139	- 86	-183	-327	-394	-405
+ 23	+ 14	+ 88	+ 55	-152	-123	-156	-100	- 54	- 52	-136	- 84	-180	-336	-392	-407
+ 37	+ 34	+ 94	+ 53	-163	-102	-164	-103	- 59	- 46	-142	- 66	-179	-324	-411	-408
+ 45	+ 24	+101	+ 55	-186	-113	-151	-105	- 52	- 52	-136	- 71	-191	-323	-413	-403
+ 64	+ 21	+ 72	+ 56	-182	-112	-165	- 99	- 44	- 41	-137	- 66	-208	-323	-419	-421
+ 80	+ 11	+ 87	+ 44	-189	-117	-141	- 77	- 39	- 50	-114	- 84	-202	-316	-410	-406
+ 68	+ 7	+ 82	+ 53	-188	-109	-133	- 85	- 59	- 62	-102	- 76	-211	-337	-415	-397
+ 53	- 5	+ 72	+ 56	-185	-104	-125	- 85	- 72	- 77	-115	- 78	-218	-321	-398	-409
+ 56	- 16	+ 64	+ 59	-186	- 90	-137	- 93	- 87	- 68	-114	- 90	-235	-331	-410	-415
+ 45	- 16	+ 54	+ 62	-195	- 89	-157	- 89	- 98	- 56	-115	-110	-262	-303	-398	-433
+ 50	- 20	+ 56	+ 72	-185	-111	-159	- 75	-106	- 56	-111	-118	-250	-315	-401	-437
+ 45	- 30	+ 69	+ 84	-189	-118	-153	- 69	-108	- 69	-114	-119	-256	-306	-404	-445
+ 53	- 14	+ 72	+ 89	-195	-136	-156	- 88	-100	- 67	- 97	-119	-238	-301	-416	-437
+ 65	- 23	+ 58	+ 95	-199	-128	-163	- 90	- 87	- 80	-107	-114	-255	-300	-403	-434
+ 58	- 8	+ 62	+ 95	-185	-135	-168	-110	- 99	- 69	-109	-128	-246	-323	-400	-434
+ 38	- 20	+ 78	+ 91	-197	-131	-175	- 96	-109	- 84	-130	-128	-241	-304	-402	-436
+ 33	- 18	+ 71	+ 81	-204	-126	-167	- 97	-119	- 78	-139	-126	-225	-292	-414	-432
+ 40	- 18	+ 86	+ 85	-213	-127	-175	-101	-129	- 80	-115	-123	-228	-300	-426	-428
+ 40	- 24	+104	+ 83	-191	-125	-180	-114	-130	- 94	- 97	-109	-223	-300	-433	-425
+ 28	- 28	+108	+ 73	-193	-127	-184	-129	-133	-104	- 95	-101	-215	-298	-444	-417
+ 25	- 26	+101	+ 79	-204	-128	-183	-128	-136	-117	- 97	- 90	-206	-313	-448	-394
+ 13	- 20	+107	+ 76	-196	-129	-182	-129	-130	-129	- 91	- 93	-224	-316	-465	-403
+ 4	- 22	+ 99	+ 74	-196	-125	-179	-115	-135	-128	- 99	- 93	-231	-317	-468	-411
+ 7	- 20	+107	+ 61	-190	-121	-170	-106	-110	-126	- 89	- 97	-233	-315	-468	-424
+ 6	- 25	+107	+ 49	-169	-131	-165	-105	-128	-119	- 83	-101	-236	-317	-491	-425
- 11	- 22	+101	+ 36	-163	-142	-167	- 93	-130	-129	- 99	-107	-239	-335	-472	-441
- 11	- 22	+ 98	+ 28	-166	-142	-167	- 72	-126	-123	- 89	-107	-246	-318	-457	-452
- 11	- 28	+ 79	+ 33	-171	-134	-159	- 80	-137	-116	- 85	-114	-240	-317	-475	-461
- 18	- 34	+ 87	+ 24	-169	-142	-153	- 74	-127	-107	- 95	-106	-236	-325	-464	-476
- 14	- 28	+ 93	+ 6	-175	-117	-166	- 77	-138	-105	- 95	-105	-232	-338	-454	-483
+ 3	- 16	+ 89	+ 8	-168	-135	-168	- 69	-139	-113	- 73	- 95	-238	-345	-465	-483
+ 10	- 13	+ 92	- 3	-156	-112	-155	- 64	-153	-107	- 79	-102	-227	-338	-459	-488
+ 11	- 15	+ 97	- 4	-152	-122	-159	- 66	-158	-104	- 77	-101	-205	-361	-449	-476
+ 25	- 18	+ 92	- 8	-158	-130	-145	- 63	-140	-112	- 67	-113	-188	-369	-456	-484
+ 19	- 21	+103	- 4	-164	-146	-160	- 58	-129	-112	- 72	-108	-179	-379	-469	-487
+ 12	- 16	+ 86	- 4	-158	-162	-151	- 54	-137	-116	- 79	-109	-184	-389	-472	-479
+ 20	- 30	+ 80	+ 7	-154	-156	-135	- 64	-140	-120	-106	-133	-183	-398	-465	-486
+ 14	- 28	+ 76	- 21	-148	-160	-138	- 59	-138	-121	-113	-143	-192	-403	-460	-484
+ 4	- 15	+ 80	- 28	-140	-155	-137	- 42	-138	-121	-110	-150	-198	-398	-458	-483
0	- 4	+ 72	- 38	-147	-142	-147	- 60	-134	-108	-123	-162	-192	-398	-470	-473
+ 5	+ 17	+ 78	- 53	-143	-128	-152	- 65	-132	-110	-118	-167	-193	-377	-472	-464
+ 5	+ 23	+ 72	- 53	-142	-108	-139	- 45	-135	-111	-110	-183	-215	-395	-471	-456
- 3	+ 31	+ 63	- 62	-125	-116	-128	- 43	-135	-111	-110	-197	-201	-396	-485	-454
- 4	+ 23	+ 79	- 65	- 99	-111	-135	- 47	-131	-129	-123	-191	-210	-386	-498	-460
- 13	+ 16	+ 86	- 70	- 92	-115	-141	- 45	-118	-118	-109	-206	-205	-371	-478	-445
- 9	+ 24	+ 74	- 61	- 92	- 95	-145	- 44	-112	-121	-120	-202	-228	-369	-479	-456
- 11	+ 14	+ 76	- 81	- 96	- 80	-123	- 65	-105	-124	-104	-202	-220	-351	-491	-462
- 13	+ 15	+ 70	- 89	- 85	- 78	-102	- 62	-101	-114	- 90	-203	-217	-349	-491	-458
- 16	+ 33	+ 65	-101	- 86	- 73	- 94	- 67	- 92	-118	- 83	-203	-226	-361	-477	-458
- 10	+ 27	+ 61	-111	- 86	- 69	-103	- 92	- 79	-119	- 83	-195	-228	-369	-463	-464
- 7	+ 22	+ 65	-124	- 88	- 75	-107	- 82	- 74	-130	- 82	-200	-238	-376	-466	-450
- 17	+ 11	+ 63	-140	- 94	- 70	-115	- 86	- 78	-105	- 85	-214	-239	-379	-466	-450
- 2	+ 18	+ 39	-142	-103	- 71	-115	- 80	- 98	-105	-107	-241	-245	-366	-446	-445
- 5	+ 24	+ 56	-147	-112	- 77	-128	- 83	-100	-104	-113	-233	-255	-366	-446	-453

TABLE I—*Continued*

EXPERIMENTAL TIME SERIES:—CUMULATIVE TOTALS OF 2399 RANDOM NUMBERS

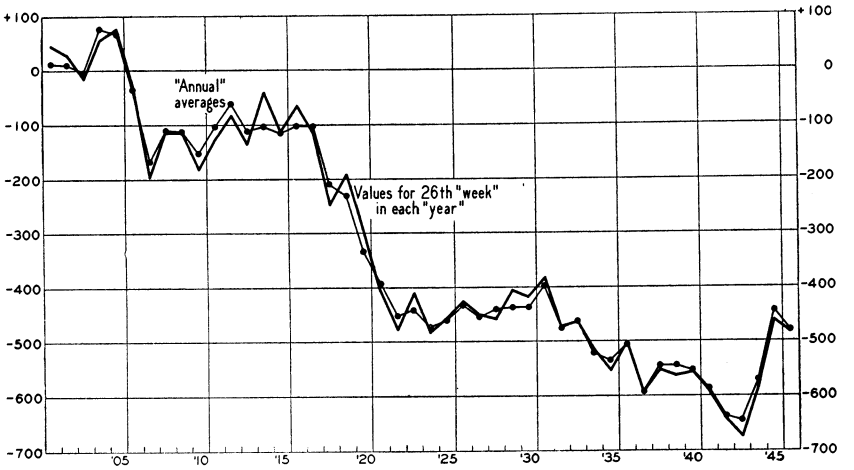
-452	-452	-454	-462	-485	-338	-495	-501	-537	-563	-552	-535	-586	-643	-598	-441
-466	-461	-454	-468	-492	-345	-509	-500	-538	-566	-556	-542	-600	-656	-592	-424
-476	-464	-451	-461	-498	-336	-505	-521	-544	-565	-567	-538	-603	-662	-596	-425
-453	-468	-462	-459	-505	-349	-503	-521	-535	-581	-558	-544	-592	-657	-596	-432
-455	-452	-463	-448	-504	-352	-482	-521	-521	-593	-553	-538	-592	-656	-595	-436
-436	-474	-460	-459	-506	-352	-488	-526	-517	-579	-560	-533	-597	-654	-586	-424
-441	-472	-472	-472	-502	-354	-482	-522	-505	-583	-572	-533	-585	-636	-580	-423
-447	-464	-471	-465	-491	-356	-473	-521	-495	-599	-561	-524	-590	-629	-565	-427
-464	-455	-462	-453	-487	-380	-457	-522	-466	-608	-560	-519	-587	-622	-574	-438
-462	-440	-442	-451	-488	-383	-476	-501	-476	-608	-572	-512	-575	-622	-601	-416
-479	-438	-432	-438	-472	-394	-471	-505	-493	-617	-582	-517	-587	-632	-586	-395
-469	-443	-428	-436	-470	-399	-465	-504	-499	-604	-584	-523	-571	-633	-588	-404
-461	-465	-409	-422	-473	-411	-460	-502	-489	-611	-581	-512	-588	-629	-577	-408
-463	-457	-427	-429	-460	-406	-457	-502	-481	-625	-578	-522	-587	-632	-572	-406
-474	-449	-418	-414	-467	-431	-455	-497	-489	-646	-573	-545	-594	-637	-583	-402
-493	-471	-413	-418	-467	-426	-457	-513	-473	-635	-559	-562	-600	-636	-573	-411
-504	-465	-408	-425	-464	-434	-454	-504	-474	-637	-554	-558	-605	-635	-584	-423
-506	-463	-426	-413	-451	-438	-448	-513	-461	-620	-530	-574	-591	-621	-580	-434
-496	-459	-437	-400	-456	-436	-446	-540	-463	-620	-526	-583	-594	-620	-583	-432
-485	-471	-436	-397	-457	-439	-468	-534	-469	-614	-524	-591	-588	-623	-583	-437
-483	-472	-453	-387	-473	-432	-480	-543	-482	-608	-508	-590	-584	-616	-572	-419
-490	-472	-443	-379	-471	-437	-484	-536	-478	-595	-527	-588	-573	-625	-570	-424
-496	-497	-426	-378	-477	-425	-498	-528	-489	-608	-537	-577	-573	-638	-560	-425
-501	-484	-424	-362	-473	-435	-489	-527	-475	-627	-521	-566	-579	-640	-543	-439
-514	-463	-440	-365	-469	-426	-498	-548	-490	-623	-527	-568	-580	-652	-535	-429
-491	-473	-456	-389	-471	-423	-496	-544	-485	-608	-513	-559	-582	-651	-516	-442
-487	-460	-451	-392	-470	-439	-492	-549	-487	-610	-517	-542	-587	-650	-504	-429
-495	-466	-457	-408	-456	-442	-486	-554	-478	-611	-518	-556	-596	-658	-506	-436
-489	-459	-454	-409	-456	-439	-492	-552	-489	-609	-521	-543	-603	-647	-500	-444
-497	-467	-462	-420	-440	-447	-487	-549	-473	-603	-525	-542	-597	-651	-499	-452
-491	-461	-459	-421	-435	-453	-469	-536	-476	-607	-541	-553	-592	-661	-493	-462
-496	-454	-465	-417	-424	-474	-459	-549	-467	-595	-537	-538	-578	-652	-505	-464
-502	-469	-462	-418	-419	-467	-457	-554	-471	-610	-528	-532	-587	-651	-487	-468
-497	-453	-470	-419	-427	-474	-454	-551	-471	-600	-533	-532	-586	-673	-473	-461
-481	-451	-463	-415	-436	-494	-449	-551	-483	-597	-537	-537	-587	-675	-484	-464
-481	-439	-464	-409	-431	-507	-466	-538	-479	-602	-543	-524	-603	-674	-481	-468
-477	-446	-468	-414	-410	-514	-479	-538	-485	-593	-547	-541	-608	-655	-482	-467
-485	-439	-454	-405	-408	-493	-480	-549	-495	-601	-535	-542	-628	-660	-479	-472
-485	-439	-453	-396	-399	-492	-465	-559	-508	-609	-548	-546	-621	-670	-481	-483
-482	-418	-460	-390	-397	-490	-469	-561	-521	-605	-548	-542	-639	-677	-479	-485
-474	-411	-450	-394	-411	-474	-456	-556	-523	-592	-545	-552	-630	-695	-472	-500
-464	-413	-451	-395	-411	-458	-453	-548	-517	-570	-526	-565	-616	-673	-473	-512
-465	-423	-461	-396	-411	-471	-453	-544	-522	-570	-535	-564	-633	-690	-473	-509
-469	-430	-481	-382	-399	-460	-439	-543	-510	-564	-539	-566	-632	-674	-472	-514
-460	-427	-467	-371	-404	-441	-433	-539	-502	-572	-530	-571	-640	-665	-462	-514
-452	-432	-458	-384	-417	-431	-434	-546	-501	-564	-527	-584	-635	-646	-449	-526
-447	-443	-469	-378	-425	-449	-439	-550	-517	-557	-526	-579	-630	-638	-436	-525
-450	-423	-468	-394	-432	-446	-445	-547	-509	-544	-543	-579	-654	-627	-438	-522
-440	-430	-474	-399	-437	-446	-446	-546	-502	-549	-544	-580	-660	-620	-432	-511
-443	-427	-497	-409	-412	-452	-443	-563	-520	-545	-555	-558	-649	-640	-425	-508
-454	-412	-502	-390	-415	-442	-433	-566	-521	-546	-561	-571	-656	-636	-428	-528
-459	-421	-509	-394	-411	-450	-439	-555	-523	-528	-555	-578	-666	-634	-430	-526
-476	-417	-497	-402	-409	-456	-442	-546	-515	-531	-563	-592	-669	-641	-431	-519
-480	-426	-474	-406	-408	-458	-448	-554	-512	-524	-571	-591	-658	-636	-436	-514
-478	-434	-490	-418	-414	-472	-444	-561	-508	-519	-572	-591	-640	-625	-440	-506
-476	-426	-501	-407	-418	-470	-465	-571	-512	-499	-554	-582	-635	-631	-444	-507
-469	-420	-496	-427	-415	-475	-465	-550	-512	-506	-555	-587	-641	-647	-451	-497
-474	-416	-496	-438	-400	-480	-467	-557	-518	-516	-547	-576	-639	-629	-445	-484
-458	-413	-487	-449	-386	-481	-477	-585	-532	-520	-547	-563	-626	-632	-454	-511
-462	-414	-483	-446	-385	-471	-474	-580	-535	-531	-542	-567	-629	-629	-447	-507
-451	-419	-462	-454	-372	-464	-481	-585	-552	-531	-561	-569	-639	-628	-458	-521
-452	-410	-453	-457	-364	-472	-497	-601	-548	-542	-562	-573	-643	-636	-452	-531
-454	-398	-457	-464	-370	-468	-491	-586	-534	-542	-559	-577	-648	-632	-462	-527
-477	-403	-459	-462	-355	-473	-477	-559	-526	-538	-557	-575	-645	-640	-462	-526
-482	-405	-457	-463	-363	-495	-487	-561	-523	-542	-551	-580	-640	-645	-456	-554
-470	-430	-453	-454	-364	-486	-470	-562	-513	-543	-552	-580	-639	-638	-463	-542
-469	-438	-450	-463	-367	-488	-479	-556	-525	-554	-560	-587	-655	-626	-461	-531
-448	-433	-448	-462	-367	-491	-478	-552	-506	-544	-550	-578	-669	-610	-462	-542
-451	-428	-430	-463	-367	-500	-477	-544	-522	-551	-543	-571	-670	-599	-467	-535
-438	-433	-440	-473	-363	-512	-472	-531	-527	-550	-545	-572	-656	-591	-453	-522
-433	-432	-435	-495	-360	-500	-471	-542	-539	-541	-542	-577	-642	-599	-443	-522
-446	-443	-453	-491	-341	-498	-477	-545	-533	-547	-541	-592	-634	-595	-449	-512
-459	-439	-460	-493	-343	-493	-478	-531	-535	-548	-543	-609	-637	-593	-438	-520
-455	-452	-465	-506	-336	-492	-481	-532	-540	-551	-542	-606	-646	-605	-445	-522
-449	-450	-466	-496	-329	-497	-497	-536	-552	-552	-531	-609	-642	-607	-432	

For the present use the 2400 4-digit numbers on the first six pages of Tippett's tables were read in order down successive columns. Because 0000 occurs once on these pages, but is discarded, only 2399 numbers were obtained for cumulation. When and if an extension of the present series is required, or a separate independent series, it may be obtained by continuing with the remainder of Tippett's tables.

Cumulation of the numbers thus derived from the tables resulted in the series given in Table I and shown graphically in the three sections

CHART II

"ANNUAL" AVERAGES AND MID-POINTS OF RANDOM-DIFFERENCE EXPERIMENTAL TIME SERIES



of Chart I. The numbers in the table are to be read in order down successive columns. The arrangement of the data in the chart as a weekly series for 46 years, and a little over, is of course wholly arbitrary, but convenient for one illustrative use to be made of the series.

To give a clearer impression of the course of the entire series than can be obtained from the separate detailed segments, 52-item "annual" averages have been plotted in Chart II. With them are shown the values of the 26th item in each successive 52-item group.[1]

The pronounced downward trend of the whole 2399-item series at once excites suspicion that the mean of the population from which the original numbers were drawn was negative rather than zero. Statistical test, however, fails to support the suspicion: the mean of the original 2399-item sample is $-.2176$, with a standard error of $+.2042$.

[1] It should be noted that these equally-spaced items may themselves be regarded as a random-difference series with differences drawn from a normal population of zero mean, but in this case of standard deviation $10\sqrt{52}$.

USES OF THE SERIES

An important application of the random-difference series here presented will be found in visual comparisons with actual time series, and probably in comparisons of appropriate statistical constants derived from the experimental random-difference series and from the actual series, as an aid in ascertaining whether and to what extent the actual series shows the characteristics of a cumulation of random changes. The desirability of examining actual time series for the presence of such characteristics has been sufficiently stressed above. It will be apparent from brief study of the curves in Chart I that an essentially random-difference series of only 200–300 items, which would be regarded as a very long annual series or a fairly long monthly series in economic statistics, might very easily be taken mistakenly to be a series dominated by a true irregular cycle with superimposed random fluctuations.

In the actual statistical analysis of time series found to have important or dominant random-difference characteristics, a number of applications of the accompanying "experimental time series" readily suggest themselves. Some students of stock and commodity prices attribute great forecasting significance to certain "formations" that appear more or less conspicuously in the charted price data—such as "resistance and support levels," "lines," and "head and shoulder formations." Other students of these prices scoff at such ideas. Valuable evidence on the probable significance of such "formations" could be obtained by ascertaining with what relative frequency they are found in a random-difference series. If they occur as frequently and as clearly in the random-difference series as in the actual series, it is to be supposed that they are without forecasting significance, for it is known that changes in a random-difference series are quite unpredictable.

Another application appeared from the reaction of a close student of stock-price behavior who was shown Chart I without identifying legends, and asked for a judgment whether it was a stock-price or a commodity-price series. After brief study he stated confidently that it was "not a stock price series." The basis for this judgment he felt lay in the fact that the random-difference series was "too jagged." This difference in characteristics at once suggested to him a feature of stock prices worthy of study for its possible forecasting value.

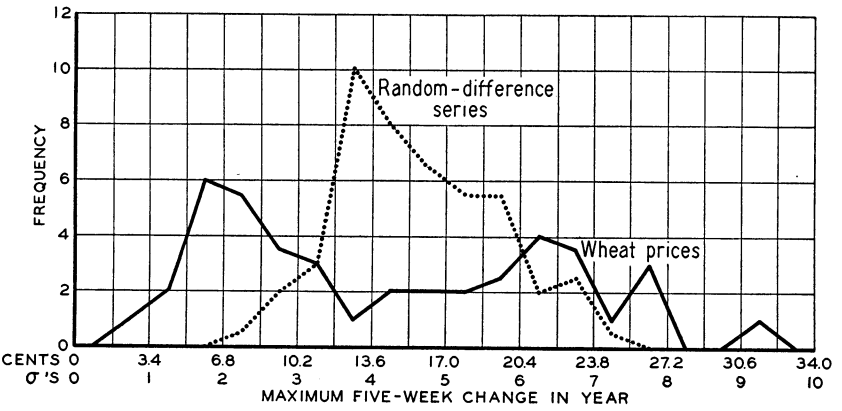
In general, applications of the experimental random-difference series in statistical practice are most likely, I suspect, to take the form of tests of statistical significance of observations that suggest uniformities in behavior-patterns of actual time series. A summary of the steps taken in one such actual test may be helpful.

Elsewhere I have shown the existence of a characteristic of wheat-price changes which I regarded as distinctly peculiar and significant.[1] When maximum price increases over intervals of five weeks or less were selected, one for each year, their frequency distribution was found to consist of two distinct, apparently slightly skewed, frequency groups, one with a mode in the neighborhood of 7 cents, the other with a mode in the neighborhood of 22 cents, as shown by the frequency polygon represented by the solid line in Chart III. The price changes were measured in terms of cents per bushel at the 1913 price level to remove

CHART III

FREQUENCY DISTRIBUTIONS OF MAXIMUM 5-WEEK CHANGES IN DEFLATED WHEAT PRICES AND OF MAXIMUM 5-INTERVAL CHANGES IN RANDOM-DIFFERENCE SERIES

(The scale in σ's applies to both series.)



differences in magnitude attributable merely to difference in the general wholesale price level in different years. In the original presentation of these findings the obvious interpretation was accepted, without critical testing, that the price increases falling in the upper group reflected the existence of surrounding circumstances differing notably, and rather sharply, from those attending smaller price increases. The study then proceeded to an investigation of price behavior in these apparently quite special cases.

The question deserves to be raised, however, whether the appearance of two distinct groups in the frequency distribution really reflects a notable characteristic of wheat prices (quite possibly to be found in some other price series also)[2] or whether it is perhaps merely a conse-

[1] In "Cycles in Wheat Prices," *Wheat Studies of the Food Research Institute*, November, 1931, VIII, 19–20.

[2] I find corn prices showing slight evidence of such a characteristic; oats prices, none.

quence of the use of a very special method of selection of the items entering into the distribution. Frequency distributions of wheat price changes selected in other more obvious ways show no such bimodality.

The method of selection used to reveal this apparent peculiarity of wheat-price increases was such that it is difficult to show deductively what sort of distribution should be expected on given assumptions.[1] It seems necessary in the circumstances to resort to testing the effects of the method of selection on a series having at least the main behavior characteristics of wheat prices. Then if the same method of selection gives significantly different results on wheat prices than on the artificial series, the difference may be accepted as evidence of a real "peculiarity" of wheat prices—that is, an evidence of a characteristic not possessed by the artificial series. The random-difference series is clearly more appropriate for such a test than a random series or a harmonic series.[2]

When the random-difference series here presented is given the same treatment as the wheat-price series, the distribution shown by the frequency polygon represented by the dotted curve in Chart III is obtained. Establishment of a comparable basis for the horizontal scales of the two frequency polygons is unnecessary to demonstrate the point under investigation: namely, that the method of selection employed does not yield a bimodal distribution when applied to the test series.[3]

In Chart III the two frequency distributions have been placed on a comparable basis by taking as a unit for measurement of magnitude the respective standard deviations of the elementary (one-week) changes in the two series. These are 10 for the experimental random-difference series, as already stated, and 3.4 cents for the wheat price series.[4] The class interval is in each case .5. Comparison of the two distributions indicates, in the light of other knowledge of wheat prices, a situation which I had previously suspected but apparently had considered too lightly: that the instances which supply the group of small wheat-price changes reflect the influence of notably peculiar conditions.[5] In fact

[1] The fact that the maximum increase recorded is sometimes over an interval of less than five weeks introduces a complication which might perhaps be disregarded, in which case the problem might be viewed as one of determining the distribution of maximum individuals in samples of 52 (52 possible 5-week changes ending in any one 12-month period). This distribution could readily be obtained from the work of Tippett ("On the Extreme Individuals and the Range of Samples Taken from a Normal Population," *Biometrika* 1925, XVII, 364–87) if the items in the samples could be regarded as independent, but of course they are not.

[2] Indeed, construction of the random-difference series was originally suggested by consideration of the requirements of this particular problem.

[3] If the difference between the two frequency distributions were of questionable significance, the χ^2 test would have been employed to determine the statistical significance of the difference.

[4] The figure of 3.4 cents is an adequate though somewhat rough approximation computed from a sample obtained by differencing regularly distributed sections of the wheat price series.

[5] These instances arise chiefly from years in which wheat prices were in the downward phase of what have called the "long cycles" in wheat prices.

they appear to reflect conditions departing more from those of the experimental series than do the instances which supply the group of large wheat-price changes.

Finally it should be noted that the series here presented cannot fulfill all the requirements that may arise for an experimental random-difference series. This series represents only one possible type of the class, though one chosen because of its probably superior generality. Other types, with some notably different features, may be obtained by derivation from random numbers drawn from populations of non-normal frequency distributions. Still other types may be obtained by varying in some systematic fashion the standard deviation of the population from which the drawings are made.[1] I find that to the important extent that wheat prices resemble a random-difference series, they resemble most closely one that might be derived by cumulating random numbers drawn from a slightly skewed population of standard deviation varying rather systematically through time.

[1] It may be questioned whether a series so drawn should be regarded as strictly random. Systematic variation of the standard deviation of the population, its mean being kept at zero, would introduce no correlations among algebraic values of the numbers drawn, but would introduce correlations among their values, signs neglected.