Econometrica, Vol. 28, 4 (October 1960)

A REVISION OF PREVIOUS CONCLUSIONS REGARDING STOCK PRICE BEHAVIOR

BY ALFRED COWLES[1]

This paper reports results which verify the general proposition that, where each unit of a time series is an average of points within that unit, the effect of such averaging will be to introduce a positive first-order serial correlation in the first differences of such a series even where the original series is a random chain. The findings here reported are the result of an investigation undertaken after Professor Holbrook Working pointed out to me that certain erroneous conclusions had resulted from failure to recognize that such a disturbance had been caused by this averaging process in one of several time series analyzed in a paper by the late Herbert E. Jones and myself, entitled "Some A Posteriori Probabilities in Stock Market Action," which was published in the July, 1937, number of *Econometrica*, pages 280–294.

IN A PAPER published in 1937 the late Herbert E. Jones and I reported an investigation as to the evidence of inertia or momentum in stock prices, our approach to the problem being as follows. In a penny-tossing series there is a probability of one half that tails will follow heads and vice versa. If the stock market rises for one day, week, month, or year, is there a probability of one half that it will decline in the succeeding comparable unit of time? In an attempt to answer this question we counted sequences and reversals, a sequence occurring when a rise follows a rise, or a decline a decline, and a reversal occurring when a decline follows a rise, or a rise a decline. We appraised the significance of the observed ratios of sequences to reversals by computing the probability that a given ratio occurred by chance from a random population. Our analysis included series for which the intervals between observations were 20 minutes, 1 hour, 1 day, 1, 2, and 3 weeks, 1, 2, 3, . . ., 11 months, and 1, 2, 3, . . ., 10 years, a total of 27 different series. For every one of the 20 series in which the time unit was less than 4 years the sequences outnumbered the reversals.

Our analysis was actually an investigation of the first-order serial correlations in the first differences of the stock price series, and Professor Holbrook Working of Stanford University has pointed out that taking monthly *averages* of daily or weekly prices will produce a positive correlation in such a series even where the original series is a random chain. He has suggested that this effect of averaging may explain the particularly high apparent predictability of monthly changes despite virtual absence of predictability in changes over three-week intervals which was noted in our 1937 paper as a puzzling phenomenon.

[1] For helpful suggestions I am indebted to Professors Holbrook Working of Stanford University and Arthur M. Okun of Yale University.

The Cowles-Jones 1937 *Econometrica* paper makes no reference to whether the data employed in the analyses there reported in Table 1 and Figure 1 were averages of time units or closing quotations. No statement is made, for example, as to whether the unit of one month was an average of daily, weekly, or high and low stock prices for the month, or whether it was an average of end-of-the-month prices. Our work sheets of 23 years ago have not been preserved and today I have no clear recollection as to details regarding the data employed. Although we were at that time aware of the general proposition that disturbance in a time series may be caused by application of an operational process, our failure to cover this point specifically in the 1937 paper indicates that we overlooked the need for taking precautions to insure elimination of the effect of averaging which Professor Working has pointed out.

As a result of a recent check, I have determined that the Standard Statistics weekly index was based on Wednesday closing prices for each week. These are the data which we used for the three series where the units of time were one, two, and three weeks, respectively.

For the series in which the time unit was one month we used a 1200-month index of rail stock prices published by Colonel Leonard P. Ayres at the Cleveland Trust Company. This was composed of five different indexes, three of them constructed at Harvard University and published in its *Review of Economic Statistics*, and two constructed by the Cleveland Trust Company which linked all five together in order to produce the continuous 1200-month index. *The Review of Economic Statistics*, 1928, states that its index covering the period from 1834 to the end of 1852 used middle-of-the-month prices, and that its index for the period from 1853 to 1865 employed beginning-of-the-month prices. I have been unable to find information on this point with regard to the data used in the Harvard index for 1866-1880. The Cleveland Trust Company, on the other hand, reports that its index covering the period from 1897 to 1935 was constructed by using the average of high and low prices for each month. It has no record as to how its index for 1881-1896 was constructed. The first 32 per cent of the 1200-month rail stock index was, therefore, constructed at Harvard in a manner consistent with our interpretation of the data, but this same interpretation is subject to the objection raised by Professor Working when applied to the last 38 per cent constructed by the Cleveland Trust Company.

Professor Working's proof of his proposition appears in an accompanying note. Although this is concerned with averaging of successive groups of items in a random chain it seems likely that the proposition also holds for the monthly mid-range used by the Cleveland Trust Company in its index of rail stock prices for 1897-1935. Professor Arthur M. Okun, who also provided a proof of Professor Working's proposition, has suggested that the matter may be viewed intuitively in the following manner. Assuming the first differences to be random, if the market rose in January, so that the index at the end of the month was higher than at the beginning, it is probable that the average of all days in January would be higher than for the first day of that month, and hence higher than the average of all days in December. Similarly the average of all days in January, being lower than for the last day of that month, would probably be lower than the average of all days in February. The converse of this proposition would be true if the market declined in January, in which case the average of all days in January would probably be lower than the average of all days in December and higher than for February.

Professor Harold T. Davis has reported an experiment in which he took a random series: $x_1, x_2, \ldots, x_n$, where $n = 1200$ and x_i was between 0 and 1. From this he constructed a second random series: $u_i = 2x_i - 1$, so that all the values of u_i would be between -1 and $+1$. From this series he constructed a third series: $y_1 = u_1$, $y_2 = y_1 + u_2$, $y_3 = y_2 + u_3$, etc. which gave a series in which the first differences were random. This series was then divided into blocks of 12 items, which thus gave a series corresponding to the 100-year monthly index of rail stock prices which was part of the data analysed in our 1937 *Econometrica* paper. Using a series corresponding to the twelfth month of each of 100 years, Professor Davis got a count of 50 sequences and 48 reversals which is almost exactly in agreement with expectation. When, however, he used the average of high and low for each block of 12 the count was 58 sequences and 40 reversals, the standard error being 5. This result indicates that averaging highs and lows for each year will introduce correlations analogous to those resulting from the averaging of prices at uniformly spaced intervals which have been noted by Professor Working.

In our 1937 paper, where the unit of time was one day, we used the Dow Jones hourly industrial averages and there is no record as to whether we averaged the hourly indexes for each day or used the close. It is probable, however, that we used the close since Dow Jones does not publish daily averages of its hourly indexes and to get such averages would have involved us in a big computing job for which there would have been no rational motive. Similarly, where the unit of time was one year, we used the 1200-month rail stock index published by the Cleveland Trust Company, and here again there is no evidence as to whether we used for each year an average of the 12 months or January, although we probably used the the latter. The assumption that in 1937 we used the close for our daily index, and January for our yearly index, is confirmed by recent computations in which the close and January were employed, respectively. As shown in

Table II on page 913, ratios of sequences to reversals in the current analyses employing time units of one day and one year are in both cases higher than those which we reported in 1937.

In view of the question raised by Professor Working, I have made new computations of sequences and reversals, in each case using figures for one specified point in each time unit, such as the close, thus avoiding the difficulty of employing the previously mentioned averages of time units. The results of these recent calculations are set forth in Table I:

TABLE I

SEQUENCES AND REVERSALS IN STOCK PRICE INDEXES

Unit	Index	Period	Actual Number of Sequences	Actual Number of Reversals	Expected Number of Sequences	Deviation of Actual from Expected Number of Sequences	Standard Deviation
1 day	Dow Jones Daily Industrial Stock Price Index[1]	August, 1955 to July, 1959	570	435	502.5	67.5	16
1 week	Standard Statistics Weekly Stock Price Index[2] Standard & Poor's Daily Industrial Stock Price Index[3]	1918–1935 1936–1958	1138	990	1064	74	23
1 month	Harvard U. Monthly Rail Stock Price Index[4] Dow Jones Daily Industrial Stock Price Index[5] Standard & Poor's Daily Industrial Stock Price Index[6]	1834–1865 1897–May, 1922 1928–1958	530	466	498	32	16
1 year	Harvard U. Monthly Rail Stock Price Index[7] Cowles Commission Standard & Poor's Monthly Industrial Stock Price Index[8]	1834–1865 1871–1958	60	51	55.5	4.5	5.2

[1] Close of each day.
[2] Wednesday's close of each week.
[3] Close for first trading day of each week.
[4] For 1834-1852 middle of each month; for 1853-1865 beginning of each month.
[5] First day of each month.
[6] Close for first day of each month.
[7] First or 15th of January of each year.
[8] Average for January of each year.

In certain cases the total of sequences and reversals reported in Table I is slightly less than the potential number, based on the period covered. This is due to a few instances in which price indexes remained unchanged for successive time units and also, in the case of the index representing time units of one month, to the fact that the New York Stock Exchange was closed during the first four months after the outbreak of World War I in the summer of 1914. The old series employing units of one week from 1918 to 1935 is included in Table I because as previously explained, its validity is unquestionable.

The new computations reported in Table I show an excess of sequences over reversals for all units of time investigated, namely one day, week, month, and year. In general, the results confirm those reported in our 1937 paper except for the series employing units of one month where the excess of sequences over reversals in my present study is substantially less than the excess reported in 1937. This confirms Professor Working's theory since for part of the 1200-month series analysed in 1937 we inadvertently employed data which manifested the effect of averaging which he has noted. Table II compares the current results reported in Table I with results reported in our 1937 paper.

TABLE II
SEQUENCES AND REVERSALS IN STOCK PRICE INDEXES

Unit	Actual Number of Sequences	Actual Number of Reversals	Expected Number of Sequences	Excess of Actual over Expected Number of Sequences	Standard Deviation
		1937 Report			
1 day	650	550	600	50	17
1 week	519	418	468.5	50.5	15
1 month	748	450	599	149	17
1 year	53	46	49.5	3.5	5
		1959 Report			
1 day	570	435	502.5	67.5	16
1 week	1138	990	1064	74	23
1 month	530	466	498	32	16
1 year	60	51	55.5	4.5	5.2

The suggestion has been made that, where the unit of time is one day, a so-called closing average, due to inactive trading, might in reality be an average for the whole day. To check this possibility, we have examined current records of the New York Stock Exchange for August 13, 14, and 17, three days in which trading volume was about 2,000,000 shares each day, or three quarters of the average daily volume of shares traded for the last five years. On two of these three days sales of 28 of the 30 stocks included in the Dow Jones Industrial Averages were made in the last 1—1/2 hours of the 5—1/2 hour trading day, and on the third day sales of all 30 stocks occurred in the last 1—1/2 hours. The average number of sales was 9 for each of these stocks during the last 1 – 1/2 hours of each day. About half of the prices which composed the closing average were for sales which took place less than 10 minutes before the end of the session, and about 90 per cent of the entire 30 stocks sold during the last half hour. The average time of final sale for all the Dow Jones Industrial stocks is about 15 minutes before the close, so that it unquestionably is a real closing average, and not an average for a large fraction of the day.

To further screen out the effect of averaging in the daily index, however, and to provide an even more stringent test of the persistence hypothesis, I made a count of sequences and reversals in the daily index of closing averages for 1009 trading days in which only every third day was counted, in each case omitting two intervening days and thereby reducing the number of observations to 336. The count here was 190 sequences and 146 reversals which almost exactly confirms the result when every trading day is counted.

For units of time shorter than one day, such as 20 minutes, or even one hour, it would not be practicable to avoid the effect of averaging because the data in these cases would necessarily be distributed over a considerable part of the short time interval between observations.

There can be no doubt as to the correctness of Professor Working's hypothesis that taking monthly averages of daily or weekly prices will produce a positive first-order serial correlation in the first differences of the series of averages, even though the original series be a random chain. Consideration of this effect of averaging invalidates conclusions based on the high ratio of sequences to reversals for the monthly series reported in the Cowles-Jones 1937 *Econometrica* paper. The results there reported for the daily, weekly, and annual series, however, have been confirmed by computations recently made in which the effect of averaging was avoided.

A positive first-order serial correlation in the first differences has been disclosed for every stock price series analysed in which the intervals between successive observations are less than four years. When allowance is made for brokerage costs, however, there is nothing in this situation to indicate that the stock exchange is not functioning as a free competitive market in which theoretically any such tendency toward correlation would be eliminated. Professor Tjalling C. Koopmans has suggested that, if the persistence in stock price movements were sufficient to provide capital gains appreciably in excess of brokerage costs, professional traders would presumably be aware of this situation and through their market operations would inadvertently wipe out the persistence in price movements from which they were attempting to profit. Whether or not this has actually occurred, the fact remains that, while our various analyses have disclosed a tendency towards persistence in stock price movements, in no case is this sufficient to provide more than negligible profits after payment of brokerage costs.

Chicago